Chasing Ramanujan Motives

VIII Encuentro Regional de
Teoría de Números

La Paloma, Rocha, Uruguay

CMI ICTP

Fernando Rodriguez Villegas, 27/10/2025

Ramanujan formulas

THEOREMS STATED BY RAMANUJAN (XI)

G. N. WATSON*.

In this paper I discuss five of the problems, numbered (3), (5), (9), (10), (11) in Ramanujan's second letter to Hardy, quoted on pp. xxviii and 352 of the *Collected Papers*.

(3)

$$1-5.\left(\frac{1}{2}\right)^5+9.\left(\frac{1.3}{2.4}\right)^5-13.\left(\frac{1.3.5}{2.4.6}\right)^5+\ldots=\frac{2}{\{\Gamma(\frac{3}{4})\}^4}.$$

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$$= \frac{1}{6}(\log 2)^3 - \frac{\pi^2}{12}\log 2 + \left(\frac{1}{1^3} + \frac{1}{3^3} + \frac{1}{5^3} + \&c\right).$$

(2) $$1 + 9\cdot\left(\frac{1}{4}\right)^4 + 17\cdot\left(\frac{1.5}{4.8}\right)^4 + 25\cdot\left(\frac{1.5.9}{4.8.12}\right)^4 + \&c = \frac{2\sqrt{2}}{\sqrt{\pi}\cdot\left\{\Gamma\left(\frac{3}{4}\right)\right\}^2}$$

(3) $$1 - 5\cdot\left(\frac{1}{2}\right)^3 + 9\cdot\left(\frac{1.3}{2.4}\right)^3 - \&c = \frac{2}{\pi}.$$

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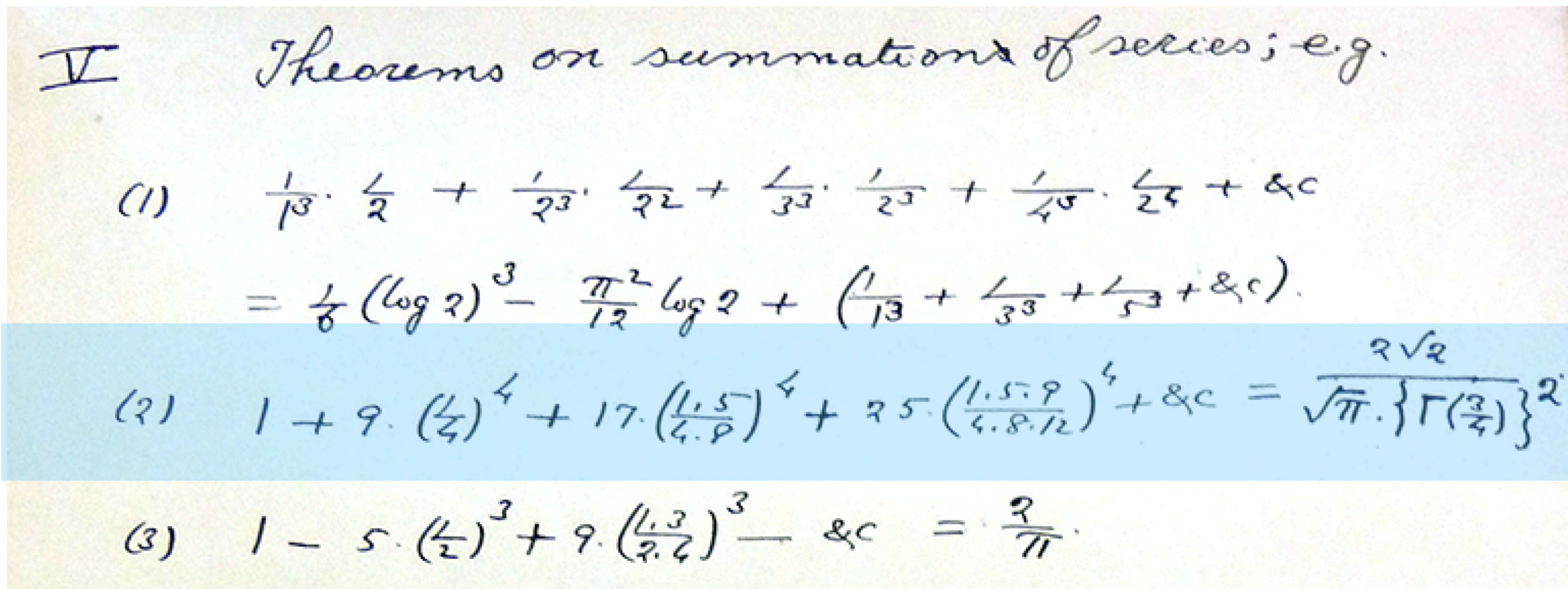

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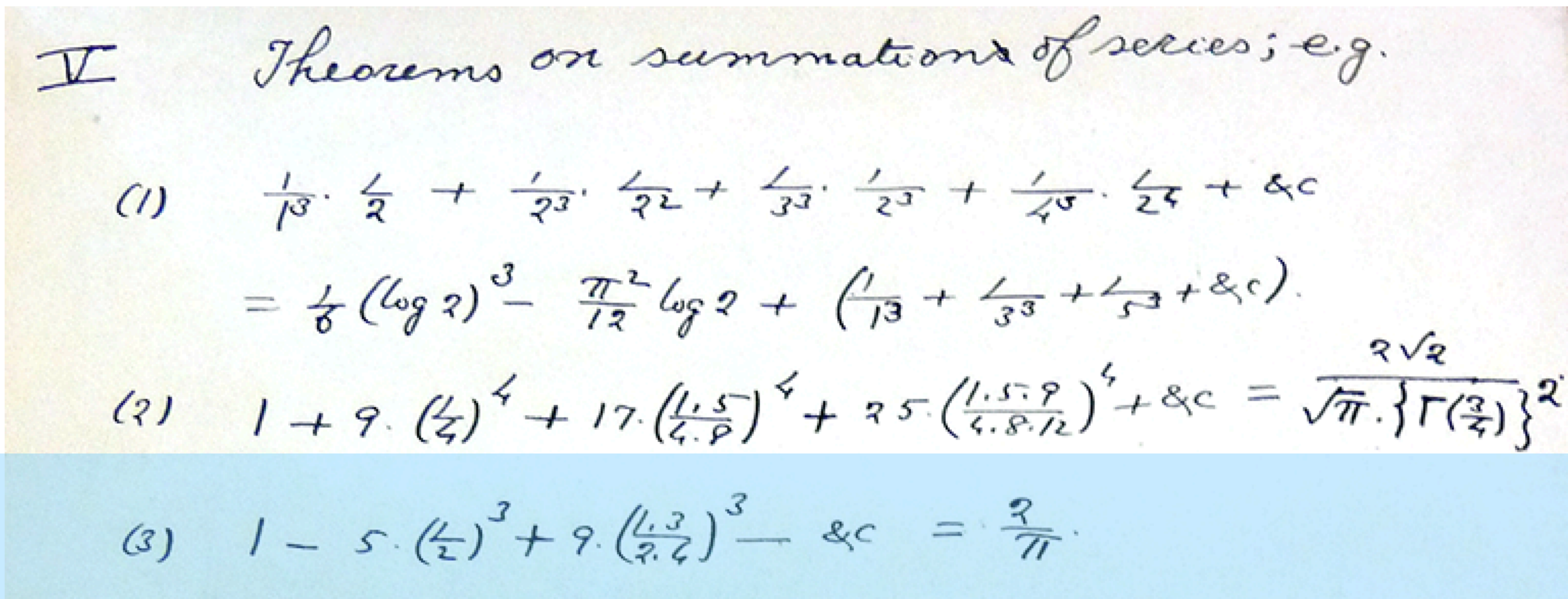

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MODULAR EQUATIONS AND APPROXIMATIONS TO π

(*Quarterly Journal of Mathematics*, XLV, 1914, 350—372)

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$$\frac{4}{\pi} = 1 + \frac{7}{4}\left(\frac{1}{2}\right)^3 + \frac{13}{4^2}\left(\frac{1\,.\,3}{2\,.\,4}\right)^3 + \frac{19}{4^3}\left(\frac{1\,.\,3\,.\,5}{2\,.\,4\,.\,6}\right)^3 + \dots,$$

$$(q = e^{-\pi\sqrt{3}},\ 2kk' = \tfrac{1}{2}), \quad \dots\dots\dots\dots\dots (28)$$

$$\frac{16}{\pi} = 5 + \frac{47}{64}\left(\frac{1}{2}\right)^3 + \frac{89}{64^2}\left(\frac{1\,.\,3}{2\,.\,4}\right)^3 + \frac{131}{64^3}\left(\frac{1\,.\,3\,.\,5}{2\,.\,4\,.\,6}\right)^3 + \dots,$$

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$$\frac{32}{\pi} = (5\sqrt{5} - 1) + \frac{47\sqrt{5} + 29}{64}\left(\frac{1}{2}\right)^3\left(\frac{\sqrt{5}-1}{2}\right)^8$$

$$+ \frac{89\sqrt{5} + 59}{64^2}\left(\frac{1\,.\,3}{2\,.\,4}\right)^3\left(\frac{\sqrt{5}-1}{2}\right)^{16} + \ldots,$$

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Sum 1. (Ramanujan)

$$\frac{1}{\pi} = \frac{\sqrt{8}}{9801} \sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4} \frac{[1103 + 26390n]}{396^{4n}}.$$

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Ramanujan, Modular Equations, and Approximations to Pi or How to compute One Billion Digits of Pi

J.M. BORWEIN, P.B. BORWEIN, AND D.H. BAILEY

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Algorithm 1. Let $\alpha_0 := 6 - 4\sqrt{2}$ and $y_0 := \sqrt{2} - 1$. Let

$$y_{n+1} := \frac{1 - (1 - y_n^4)^{1/4}}{1 + (1 - y_n^4)^{1/4}}$$

and

$$\alpha_{n+1} := (1 + y_{n+1})^4 \alpha_n - 2^{2n+3} y_{n+1}(1 + y_{n+1} + y_{n+1}^2).$$

Then

$$0 < \alpha_n - 1/\pi < 16 \cdot 4^n e^{-2 \cdot 4^n \pi}$$

and α_n converges to $1/\pi$ *quartically* (that is, with order four).
[One hundred billion digits ...]

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The series

$$\varpi(t) := \sum_{n\geq 0} a_n t^n$$

is a period function

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where ω, γ are a differential form an cycle, respectively, on a family of algebraic varieties

$$Z_t, \qquad t \in \mathbb{P}^1$$

Ramanujan-type formulas

About a New Kind of Ramanujan-Type Series

Jesús Guillera

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$$\sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{1}{2}\right)_n^5}{n!^5 2^{10n}} (820n^2 + 180n + 13) = \frac{128}{\pi^2}, \qquad (1\text{–}1)$$

$$\sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}\right)_n^3 \left(\frac{1}{4}\right)_n \left(\frac{3}{4}\right)_n}{n!^5 2^{4n}} (120n^2 + 34n + 3) = \frac{32}{\pi^2}, \qquad (1\text{–}2)$$

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Boris Gourevitch [Gourevitch 02] has sent me, by email, the formula below for $1/\pi^3$. He has found it by using *integer relations algorithms*:

$$\sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}\right)_n^7}{n!^7 \, 2^{6n}} (168n^3 + 76n^2 + 14n + 1) = \frac{32}{\pi^3}. \qquad (4\text{–}1)$$

$$\sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}\right)_n^7 \left(\frac{1}{4}\right)_n \left(\frac{3}{4}\right)_n}{(1)_n^3} (43680n^4 + 20632n^3 + 4340n^2 + 466n + 21) \left(\frac{1}{4096}\right)^n = \frac{2048}{\pi^4}$$

discovered by J. Cullen

We will focus on the following

$$\varpi_r(t) := \sum_{n\geq 0} \left(\frac{(1/2)_n}{n!}\right)^r t^n, \qquad |t| < 1$$

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1) $$\varpi_r(4^r\lambda) = \frac{1}{(2\pi i)^r} \int_{|x_i|=1} \frac{1}{1-\lambda(1+x_1)(1+x_1^{-1})\cdots(1+x_r)(1+x_r^{-1})} \frac{dx_1}{x_1} \cdots \frac{dx_r}{x_r}$$

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3) $$Z_\lambda : \qquad 0 = 1 - \lambda(1 + x_1)(1 + x_1^{-1}) \cdots (1 + x_r)(1 + x_r^{-1})$$

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By the residue theorem in say x_r

$$\varpi(t) = \frac{1}{(2\pi i)^{r-1}} \int_\gamma \omega$$

where ω is an r-1 differential form defined over $\mathbb{Q}$.

The Ramanujan-type series is

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Hence if the period is a power of π it must be

$$\frac{1}{\pi^{\frac{1}{2}(r-1)}}$$

Hodge structure

For X smooth and projective: Hodge decomposition

$$H^n_{dR}(X) \otimes \mathbb{C} = \oplus H^{pq}(X)$$

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Define the Hodge number as

$$h^{p,q} := \dim H^{p,q}$$

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Hodge vector

$$h = (1, 1)$$

For an Artin L-function of degree d

$$h = (d)$$

For a modular form of weight k

$$h = \overbrace{(1, 0, \ldots, 0, 1)}^{k}$$

For (a smooth compactification of) Z_t

$$h = \overbrace{(1, 1, \ldots, 1)}^{r}$$

Generically, this Hodge structure is irreducible

but might split at special values of the parameter

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For example, for r odd it might split as

$$h = (1, 1, \ldots, 1) = (1, 1, \ldots, 0, \ldots, 1, 1) \oplus (0, \ldots, 0, 1, 0, \ldots, 0)$$

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$$(42\partial + 5)\varpi = \frac{16}{\pi}, \qquad \partial := \frac{td}{dt}$$

For the first page example

$$1-5\cdot\left(\frac{1}{2}\right)^5+9\cdot\left(\frac{1\cdot 3}{2\cdot 4}\right)^5-13\cdot\left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^5+\ldots=\frac{2}{\{\Gamma(\frac{3}{4})\}^4}$$

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Newform orbit 16.3.c.a

Introduction
Overview Random
Universe Knowledge
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Classical Maass
Hilbert Bianchi
Siegel
Varieties
Elliptic curves over $\mathbb{Q}$
Elliptic curves over $\mathbb{Q}(\alpha)$
Genus 2 curves over $\mathbb{Q}$

Newspace parameters

Show commands: Magma / Pari/GP / SageMath

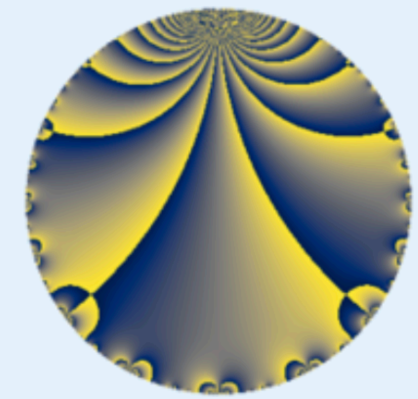
Level:	N	$=$	$16 = 2^4$
Weight:	k	$=$	3
Character orbit:	$[\chi]$	$=$	16.c (of order 2, degree 1, minimal)

Newform invariants

Self dual:	yes
Analytic conductor:	0.435968422976
Analytic rank:	0
Dimension:	1
Coefficient field:	$\mathbb{Q}$
Coefficient ring:	$\mathbb{Z}$
Coefficient ring index:	1
Twist minimal:	yes
Sato-Tate group:	$\mathrm{U}(1)[D_2]$

Properties

Label	16.3.c.a



Level	16
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Related objects

Newform orbit 16.3.c.a

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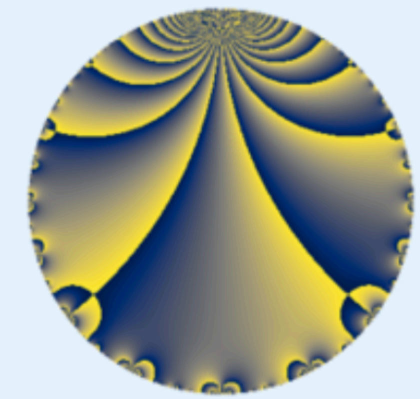

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Related objects

q-expansion

$$f(q) = q - 6q^5 + 9q^9 + 10q^{13} - 30q^{17} + 11q^{25} + 42q^{29} - 70q^{37} + 18q^{41} - 54q^{45} + 49q^{49} + 90q^{53} - 22q^{61} - 60q^{65} - 110q^{73} + 81q^{81} + 180q^{85} - 78q^{89} + \cdots + 130q^{97} + O(q^{100})$$

Expression as an eta quotient

$$f(z) = \eta(4z)^6 = q\prod_{n=1}^{\infty}(1 - q^{4n})^6$$

Griffiths transversality

Applying $\partial := \frac{td}{dt}$ shifts the Hodge type at least one step to the right

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The differential form ω has Hodge type (4,0)

Hence to reach type (3,1) for

$$(0, 1, 0, 1, 0)$$

we need δ of degree 1